XXI. *New Fundamental Experiments upon the Collision of Bodies. By Mr.* John Smeaton, *F. R. S. in a Letter to Sir* Joſeph Banks, *Bart. P. R. S.*

Read April 18, 1782.

TO SIR JOSEPH BANKS, BART. P. R. S.

S I R,

THE ſubjects of the incloſed tract have been the object of my conſideration for many years paſt; and as they contain ſome matters that have not only been variouſly reaſoned about, but variouſly concluded upon; if what is contained therein ſhall appear of ſuch a nature as either to eſtabliſh truth, as it appears to me; or to prompt ſome more able perſon, in reviewing the ſubject, to ſhew what links in my chain of reaſoning thereon are defective, ſo as to eſtabliſh the whole doctrine of moving bodies upon one plain conſiſtent baſis, my end will be equally anſwered in offering them to you, to be laid before the Royal Society, in caſe you ſhall think that the importance of the ſubject ſhall merit the ſame: furthermore, I hope to be forgiven, if in ſome parts of this paper I have expreſſed myſelf with more pointedneſs than I might have done, for I declare, that it was ſolely owing to my earneſtneſs that the ſubject of mechanic motions and powers ſhould be fully and freely inveſtigated, and eſtabliſhed upon grounds that ſhall be uncontrovertible.

I have the honour to be, &c.

Gray's-Inn,
April 18, 1782.

IT is univerſally acknowledged, that the firſt ſimple principles of ſcience cannot be too critically examined, in order to their being firmly eſtabliſhed; more eſpecially thoſe which relate to the practical and operative parts of mechanics, upon which much of the active buſineſs of mankind depends. A ſentiment of this kind occaſioned my tract upon *Mechanic Power*, which was publiſhed in the Philoſophical Tranſactions, vol. LXVI. for the year 1776. What I have now to offer was intended as a ſupplement thereto, and the experiments were then, in part, tried; but the completion thereof was deferred at that time, partly from want of leiſure; partly to avoid too great a length of the paper itſelf; and partly to avoid the bringing forward too many points at once. My preſent purpoſe is to ſhew, that the true doctrine of the *colliſion of bodies* hangs as it were upon the ſame hook, as the doctrine of the gradual generation of motion from reſt, conſidered in that paper; that is, that whether bodies are put into gradual motion, and uniformly accelerated from reſt to any given velocity; or are put in motion, in an inſtantaneous manner, when bodies of any kind ſtrike one another; the motion, or ſum of the motions produced, has the ſame relation to mechanic power therein defined, which is neceſſary to produce the motion deſired. To prove this, and at the ſame time to ſhew ſome capital miſtakes in principle, which have been *aſſumed* as indiſputable truths by men of great learning, is the reaſon of my now purſuing the ſame ſubject.

I do not mean to point out the particular miſtakes which have been made by particular men, as that would lead me into too great a length: I ſhall therefore content myſelf with obſerving, that the laws of colliſion, which have been inveſtigated by mathematical philoſophers, are principally of three kinds; *viz.* thoſe relating to bodies perfectly *elaſtic*; to bodies

perfectly

perfectly unelaftic, and perfectly *foft*; and to bodies perfectly unelaftic, and perfectly *hard*. To avoid prolixity, I fhall confider in each, only the fimple cafe of two bodies which are equal in weight or quantity of matter ftriking one another. Refpecting thofe which are perfectly elaftic, it is univerfally agreed, that when two fuch bodies ftrike one another, no motion is loft; but that in all cafes, what is loft by one is acquired by the other: and hence, that if an elaftic body in motion ftrikes another at reft, upon the ftroke the former will be reduced to a ftate of reft, and the latter will fly off with an equal velocity.

In like manner, if a non-elaftic *foft* body ftrikes another at reft, they neither of them remain at reft, but proceed together from the point of collifion with exactly one half of the velocity that the firft had before the ftroke; this is alfo univerfally allowed to be true, and is fully proved by every good experiment upon the fubject.

Refpecting the third fpecies of body, that is, thofe that are non-elaftic, and yet perfectly hard; the laws of motion relating to them, as laid down by one fpecies of philofophers, have been rejected by another; the latter alledging, that there are no fuch bodies to be found in nature whereon to try the experiment; but thofe who have laid down and affigned the doctrine that would attend the collifion of bodies of this kind (if they could be found) have univerfally agreed, that if a non-elaftic *hard* body was to ftrike another of the fame kind at reft, that, in the fame manner as is agreed concerning non-elaftic foft bodies, they neither of them would remain at reft, but would in like manner proceed from the point of collifion, with exactly one half of the velocity that the firft had before the ftroke: in fhort, they lay it down as a rule attending all non-elaftic bodies, whether hard or foft, that the velocity after the ftroke will

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be the fame in both, *viz. one half* of the velocity of the origi-
nal ftriking body.

Here is therefore the affumption of a principle, which in
reality is proved by no experiment, nor by any fair deduction
of reafon that I know of, *viz.* that the velocity of non-elaftic
hard bodies after the ftroke muft be the fame as that refulting
from the ftroke of non-elaftic *foft* bodies; and the queftion
now is, whether it is true or not?

Here it may be very properly afked, what ill effects can
refult to practical men, if philofophers fhould reafon wrong
concerning the effects of what does not exift in nature, fince
the practical men can have no fuch materials to work upon, or
misjudge of? But it is anfwered, that they who infer an equa-
lity of effects between the two forts, may from thence be
mifled themfelves, and in confequence miflead practical men
in their reafonings and conclufions concerning the fort with
which they have abundant concern, to wit, the non-elaftic
foft bodies, of which water is one, which they have much to
do with in their daily practice.

Previous to the trying my experiment on mills I never had
doubted the truth of the doctrine, that the fame velocity re-
fulted from the ftroke of both forts of non-elaftic bodies; but
the trial of thofe experiments made me clearly fee at leaft the
inconclufivenefs, if not the falfity of that doctrine: becaufe I
found a refult which I did not expect to have arifen from either
fort; and for the which, when it appeared from experiment, I
could fee a fubftantial reafon why it fhould take place in one
fort, and that it was impoffible that it could take place in the
other; for if it did, the bodies could not have been perfectly
hard, which would be contrary to the hypothefis. Of this
deduction I have given notice in my faid tract on mills, pub-
lifhed

lifhed in the Philofophical Tranfactions, vol. LI. for the year 1759 *.

It may alfo be faid, that fince we have no bodies perfectly elaftic, or perfectly unelaftic and *foft*, why fhould we expect and bodies perfectly unelaftic and *hard*? Why may not the effects be fuch as fhould refult from a fuppofition of their being *imper-fectly elaftic* joined with their being *imperfectly hard*? But here I muft obferve, that the fuppofition appears to be a contra-diction in terms.

We have bodies which are fo nearly perfectly elaftic, that the laws may be very well deduced and confirmed by them; and the fame obtains with refpect to non-elaftic *foft* bodies; but con-cerning bodies of a mixed nature, which are by far the greateft number, fo far as they are wanting in elafticity, they *are foft*, and *bruife, yield*, or *leave a mark* in collifion; and fo far as they are not perfectly foft they are elaftic, and obferve a mix-ture of the law relative to each; but imperfectly elaftic bodies, imperfectly hard come in reality under the *fame defcription* as the former mixed bodies: for fo far as they are imperfectly hard they are foft, and either *bruife* and *yield*, or leave a mark in the ftroke; and fo far as they want perfect elafticity, they are non-elaftic; that is to fay, they are bodies imperfectly elaftic, and imperfectly foft; and in fact I have never yet feen any bodies but what come under this defcription. It feems, therefore, that refpecting the *hardnefs* of bodies they differ in degrees of it, in proportion as they have a greater degree of tenacity or cohefion; that is, are further removed from perfect

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* " The effect, therefore, of overfhot wheels, under the fame circumftance of
" quantity and fall, is at a medium double to that of the underfhot: and as a
" confequence thereof, that *non-elaftic bodies, when acting by their impulfe or collifion,*
" *communicate only a part of their original power*; the other part *being fpent in*
" *changing their figure in confequence of the ftroke.*" Phil. Tranf. vol. LI. p. 133.

foftnefs,

foftnefs, at the fame time that their elaftic fprings, fo far as they reach, are very ftiff; and hence we may (by the way) conclude, that the fame mechanic power that is required to change the figure in a *fmall degree* of thofe bodies that have the popular appellation of *hard bodies*, would change it in a *great degree* in thofe bodies that approach towards foftnefs, by having a fmall degree of tenacity or cohefion. In the former kind we may rank the harder kinds of *caft iron*, and in the latter, *foft tempered clay*.

While the philofophical world was divided by the difpute about the *old* and *new opinion*, as it was called, concerning the powers of bodies in motion, in proportion to their different velocities : thofe who held the old opinion contending, that it was as the velocity *fimply*, afked thofe of the new, How, upon their principles, they would get rid of the conclufions arifing from the doctrine of unelaftic perfectly hard bodies? They replied, They found no fuch bodies in nature, and therefore did not concern themfelves about them.   On the other hand, thofe of the new opinion afked thofe of the old, How they would account for the cafe of non-elaftic foft bodies, where, according to them, the whole motion loft by the ftriking body was retained in the two after the ftroke (the two bodies moving together with the half velocity), though the two non-elaftic bodies had been bruifed and changed their figure by the ftroke ; for, if no motion was loft, the change of figure muft be an effect without a caufe? To obviate this, thofe of the old opinion ferioufly fet about proving, that the bodies might change their figure, without any lofs of motion in either of the ftriking bodies.

Neither of thefe anfwers have appeared to me fatisfactory, efpecially fince my mill experiments : for with refpect to the firft, it is no proper argument to urge the impoffibility of find-

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ing the proper material for an experiment, in anfwer to a con-
clufion drawn from an abftract idea. On the other hand, if it
can be fhewn, that the figure of a body can be changed, with-
out a *power*, then, by the fame law, we might be able to make
a *forge hammer* work upon a mafs of foft iron, without any
other power than that neceffary to overcome the friction, re-
fiftance, and original *vis inertiæ*, of the parts of the machine
to be put in motion : for, as no progreffive motion is given the
mafs of iron by the hammer (it being fupported by the anvil),
no power can be expended that way ; and if none is loft to
the hammer from changing the figure of the iron, which is
the only effect produced, then the whole power muft refide in
the hammer, and it would jump back again to the place from
which it fell, juft in the fame manner as if it fell upon a body
perfectly elaftic, upon which, if it did fall, the cafe would really
happen : the power, therefore, to work the hammer would
be the fame, whether it fell upon an elaftic or non-elaftic body ;
an idea fo very contrary to all experience, and even apprehen-
fion, of both the philofopher and vulgar artift, that I fha
here leave it to its own condemnation.

As nothing, however, is fo convincing to the mind as expe-
riments obvious to the fenfes, I was very defirous of contriving
an experiment in point ; and as I faw no hopes of finding mat-
ter to make a *direct* experiment, I turned my mind towards an
indirect one ; fo circumfcribed, however, as to prove incon-
teftably, that the refult of the ftroke of two non-elaftic per-
fectly hard bodies could not be the fame as would refult from
the collifion of two foft ones ; that is, if it can be *bona fide*
proved, that *one half* of the original power is loft in the ftroke
of foft bodies by the change of figure (as was very ftrongly
fuggefted by the mill experiments); then fince no fuch lofs

can happen in the collifion of bodies perfectly hard, the refult and confequence of fuch a ftroke muft be *different.*

The confequence of a ftroke of bodies perfectly hard, but void of elafticity, muft doubtlefs be different from that of bodies perfectly *elaftic*: for having no fpring the body at reft could not be driven off with the velocity of the ftriking body, for that is the confequence of the action of the fpring or elaftic parts between them, as will be fhewn in the refult of the experiments; the ftriking body will therefore not be ftopped, and as the motion it lofes muft be communicated to the other, from the equality of action and re-action, they will both proceed together, with an equal velocity, as in the cafe of non-elaftic foft bodies: the queftion, therefore, that remains is, what that *velocity muft be?*—It muft be greater than that of the non-elaftic foft bodies, becaufe there is no mechanical power loft in the ftroke. It muft be lefs than that of the ftriking body, becaufe, if equal, inftead of a *lofs* of motion by the collifion, it will be doubled. If, therefore, non-elaftic foft bodies lofe half their motion, or mechanical power, by change of figure in collifion, and yet proceed together with half the velocity, and the non-elaftic hard bodies can lofe *none* in any manner whatever; then, as they muft move together, their velocity muft be fuch as to preferve the equality of the mechanic power, *unimpaired,* after the ftroke the fame as it was before it.

For example, let the velocity of the ftriking body before the ftroke be 20, and its mafs or quantity of matter 8; then, according to the rule deduced from the experiments in the tract on *Mechanic Power* (fee exp. third and fourth) that power will be expreffed by 20 × 20 = 400, which × 8 = 3200; and if half of it is loft in the ftroke, in the cafe of non-elaftic foft bodies, it will be reduced to 1600; which ÷ 16 the double quantity of matter, will give 100 for the fquare of their velocity; the

fquare

fquare root of which being 10, will be the velocity of the two non-elaftic foft bodies after the ftroke, being juft one half of the original velocity, as it is conftantly found to be. But in the non-elaftic hard bodies, no power being loft in the ftroke, the mechanic power will remain after it, as before it, = 3200; this, in like manner, being divided by 16, the double quantity of matter, will give 200 for the fquare of the velocity, the fquare root of which is 14. 14, &c. for their velocity after the ftroke, which is to 10, the velocity of the non-elaftic foft bodies after the ftroke, as the fquare root of 2 to 1 ; or as the diagonal of a fquare to its fide.

It remains, therefore, now to be proved, that precifely half of the mechanic power is *loft* in the collifion of non-elaftic foft bodies ; for which purpofe my mind fuggefted the following reflections. In the collifion of elaftic bodies the effect feemingly inftantaneous, is yet performed in *time* ; during which time the natural fprings refiding in elaftic bodies, and which conftitute them fuch, are bent or forced, till the motion of the ftriking body is divided between itfelf and the body at reft ; and in this ftate the two bodies would then proceed together, as in the cafe of non-elaftic foft bodies ; but as the fprings will immediately reftore themfelves in an equal time, and with the fame degree of *impulfive force*, wherewith they were bent in this re-action, the motion that remained in the ftriking body will be totally deftroyed, and the total exertion of the two fprings, communicated to the original refting body, will caufe it to fly off with the fame velocity wherewith it was ftruck.

Upon this idea, if we could conftruct a couple of bodies in fuch a way that they fhould either act as bodies perfectly elaftic ; or, that their fprings fhould at pleafure be hooked up, retained, or prevented from reftoring themfelves, when at their extreme degree of bending ; and if the bodies under thefe

circumftances obferved the laws of collifion of non-elaftic foft bodies, then it would be proved, that one half of the mechanical power, refiding in the ftriking body, would be loft in the action of collifion ; becaufe the impulfive force or power of the fpring in its reftitution being cut off, or fufpended from acting, which is equal to the impulfive force or power to bend it (and which alone has been employed to communicate motion from one body to the other), it would make it evident, that one half of the impulfive force is loft in the action, as the other half remains *locked up* in the fprings. It alfo follows, as a *collateral circumftance*, that be the impulfive power of the fprings what it may from firft to laft, yet as one half of the *time* of the action is by this means cut off, in this fenfe alfo it will follow, that one half of the mechanic power is deftroyed ; or rather, in this cafe, remains locked up in the fprings, capable of being *re-exerted* whenever they are fet at liberty, and of producing a frefh mechanical effect, equivalent to the motion or mechanical power of the two non-elaftic foft bodies after their collifion.

Hence we muft infer, that the quantity of mechanical power expended in difplacing the parts of non-elaftic foft bodies in collifion, is exactly the fame as that expended in bending the fprings of perfectly elaftic bodies ; but the difference in the ultimate effect is, that in the non-elaftic foft bodies, the power taken to difplace the parts will be totally loft and deftroyed, as it would require an equal mechanic power to be raifed a-frefh, and exerted in a contrary direction to reftore the parts back again to their former places ; whereas, in the cafe of the elaftic bodies, the operation of half the mechanic power is, as obferved already, only locked up and fufpended, and capable of being re-exerted without a further original acceffion.

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Thefe ideas arofe from the refult of the experiments tried upon the machine defcribed in my faid tract upon Mechanic Power, and were alfo communicated to my very worthy and ingenious friend WILLIAM RUSSELL, Efq. F. R. S. at the fame time that I fhewed him thofe experiments in 1759; but the mode of putting this matter to a full and fair mechanical trial has fince occurred; and though fome rough trials, fufficient to fhew the effect, were made thereon, prior to the offering the paper on mechanical power to the Society in 1776, yet the machine itfelf I had not leifure to complete to my fatisfaction till lately; which I mention to apologize for the length of time that thefe fpeculations have taken in bringing forward.

DESCRIPTION OF THE MACHINE FOR COLLISION.

Fig. 1. fhews the front of the machine as it appears at reft when fitted for ufe.

A is the pedeftal, and AB the pillar, which fupports the whole, C, D are two compound bodies of about a pound weight each, but as nearly equal in weight as may be. Thefe bodies are alike in conftruction, which will be more particularly ex-plained by fig. 2. Thefe bodies are fufpended by two white fir rods of about half an inch diameter *ef* and *gh*, being about four feet long from the point of fufpenfion to the center of the bodies; and their fufpenfion is upon the crofs-piece II, which is mortoifed through, to let the rods pafs with perfect freedom; and they hang upon two fmall plates filed to an edge on the under fide, and pafs through the upper part of the rods. Their centers are at *k* and *l*, and the edges being let into a little notch, on each fide the mortoife, the rods are at liberty to vibrate freely upon their refpective points (or rather edges) of fufpenfion, and are determined to one plain of vibration. MN is a flat arch of white wood, which may be covered with

paper,

Paper, that the marks thereupon may be the more confpicuous.

The crofs-piece II is made to project fo far before the pillar, that the bodies in their vibrations may pafs clear of it, without danger of ftriking it ; and alfo the arch MN is brought fo far forward as to leave no more than a clearance, fufficient for the rods to vibrate freely without touching it.

Fig. 2. Shews one of the compound bodies, drawn of its full fize. AB is a block of wood, and about as much in breadth as it is reprefented in height, through a hole in which the wood rod CC paffes, and is fixed therein.

DB reprefents a plate of lead about three-eighths of an inch thick, one on each fide, fcrewed on by way of giving it a competent weight. *d*B*efg* reprefents the edge of a fpringing plate of brafs, rendered elaftic by hard hammering ; it is about five-eighths of an inch in breadth, and about one-twentieth of an inch thick. It is fixed down upon the wooden block at its end *d*B by means of a bridge plate, whofe end is fhewn *hi*, and is fcrewed down on each fide the fpring plate by a fcrews which being relaxed the fpring can be taken out at pleafure, and adjufted to its proper fituation. *kl* is a light thin flip of a plate, whofe under edge is cut into teeth like a fine faw or ratchet, and is attached to the fpring by a pin at *k*, which paffes through it, and alfo through a fmall ftud rivetted into the back part of the fpring, and upon which pin, as a center, it is freely moveable.

mn fhews a fmall plate or ftud feen edgeways raifed upon the bridge plate, through an hole in which ftud the ratchet paffes ; and the lower part of the hole is cut to a tooth fhaped properly to catch the teeth of the ratchet, and retain it together with the fpring at any degree to which it may be fuddenly bent ; and for this intent it is kept bearing gently downward, by means of a wire-fpring *opq*, which is in reality double, the

bearing

bearing part at *o* being femi-circular; from which branching off on each fide the rod *cc*, paffes to *p*, and fixes at each end into the wood at *q*. However, to clear the ratchet, which is neceffarily in the middle as well as the rod, the latter is per-forated; and alfo the block is cut away, fo far as to fet the main fpring at *e* free of all obftacles that would prevent its play from the point B. The part *fg* is fhewn thicker than the reft, by being covered with thin kid leather tight fowed on, to pre-vent a certain jarring that otherwife takes place on the meeting of the fprings in collifion.

Let us now return to fig. 1. the marks upon the arch MN are put on as follows. *op* is an arch of a circle from the center *l*, and *qr* an arch of a circle from the center *k* interfecting each other at *s*. Now the middle line of the marks *t*, *v*, are at the fame diftance from the middle line at *s* that the centers *kl* are; fo that when each body hangs in its own free pofition, with-out bearing againft the other, the rod *ef* will cover the mark at *t*, and the rod *gh* will cover the mark at *v*. From the point S upon the arches S*p* and S*q* refpectively, fet off points at an equal and competent diftance from S each way, which will give the middle of the mark *w* and *x*: and upon the arch S*p* find a middle point between the mark *v* and *w*, which let be *y*; and on the other fide, in like manner, upon the arch S*q* find a middle point for the mark *z*; then fet off the diftance S*v* or S*t* from *y* each way, and from *z* each way; and from thefe points, drawing lines to the refpective centers *l* and *k*, they will give the place and pofition of the marks *a*, *b*, and *c*, *d*; and thus is the machine prepared for ufe.

FOR TRIALS ON ELASTIC BODIES.

For this ufe take out the pins and ratchets from each refpec-tively, and the fprings being then at liberty, with a fhort bit

of ſtick (ſuppoſe the ſame ſize as the rods) turn aſide the rod *gh* with the right-hand, carrying the body D upwards till the ſtick is upon the mark *w*, as ſuppoſe at ⊙; there hold it, and with the left ſet the body C perfectly at reſt; in which caſe the rod *ef* will be over the mark *t*; then ſuddenly withdraw the ſtick, in the direction that the rod *gh* is to follow it, and the ſpring of the body D, impinging upon that of the body C, they will be both bent, and alſo reſtored; and the body C will fly off, and mount till its rod *ef* covers the mark *x*; the rod of the ſtriking body D remaining at reſt upon its proper mark of reſt *v*, till the body C returns, when the body D will fly off in he ſame manner; the two bodies thus rebounding a number of times, loſing a part of their vibration each time; but ſo nearly is the theory of elaſtic bodies fulfilled hereby, that the ſingle advantage of originally puſhing the rod *gh* beyond the mark *w*, by the thickneſs of the ſtick, or its own thickneſs, is ſufficient to carry the rod of the quieſcent body C completely to its mark *x*.

There are ſeveral other experiments that may be made with this apparatus, in confirmation of the doctrine of the colliſion of elaſtic bodies; which being univerſally agreed upon, and well known, it is needleſs further to dwell upon here; but reſpecting the application to non-elaſtic ſoft bodies, it is far more difficult to come at a fitneſs of materials for this kind of experiments, than it is for thoſe ſuppoſing perfectly elaſticity. The concluſions, however, may be attained with equal certainty.

FOR TRIALS ON NON-ELASTIC SOFT BODIES.

For this purpoſe the ratchets muſt be applied and put in order as before deſcribed, and the ſprings being both put to their point of reſt, let the body D be put to its mark *w* in the

same manner as before defcribed, and the body C to reft. The body D being let go, and ftriking the body C at reft, in confequence of the ftroke, the fprings being hooked up by the ratchets, they both move from their refting marks *t*, *v*, refpectively toward M: Now if they both moved together, and the rod *ef* covered the mark *c*, and the rod *gh* covered the mark *d* at their utmoft limit, then they would truly obey the laws of non-elaftic foft bodies; becaufe their medium afcent would be to the mark *z*, which is juft half the angle of afcent to the mark *x*; but as in this piece of machinery, though the main or principle fprings are hooked up, yet every part of them, and all the materials of which they are compofed, and to which they are attached, have a degree, or more properly fpeaking, a certain compafs of *elafticity*, which, as fuch, is perfect, and no motion loft thereby.

We muft not, therefore, expect the two compound bodies after the ftroke to ftick together without feparating, as would be the cafe with bodies truly non-elaftic and foft; but that from the elafticity they are pofleffed of, they will by rebounding be feparated; but that elafticity being perfect, can occafion no lofs of motion to the fum of the two bodies; fo that if the body C afcends as much above its mark *c* as the body D falls fhort of its mark *d*, then it will follow, that their medium afcent will ftill be to the mark *z*, as it ought to have been, had theybeen truly non-elaftic foft bodies; and this, in reality, is truly the cafe in the experiment, as nearly as it can be difcerned.

After a few vibrations, by the rubbing of the fprings againft one another, they are foon brought to reft; and here they would *always reft* had they been truly and properly perfect non-elaftic foft bodies; but here, as in the cafe of thefe bodies, by a change of the figure and fituation of the component parts, there is expended one half of the mechanical power

of

of the firft mover, yet in this cafe the other half is not *loft*, but *fufpended*, ready to be re-exerted whenever it is fet at liberty ; and that it is really and *bona fide one half* and neither more or lefs, appears from this uncontroverted fimple principle, that the power of reftitution of a perfect fpring is exactly equal to the power that bends it. And this may, in a certain degree, be fhewn to be fact by experiment, if there were any need of fuch a proof ; for if, when the bodies are at reft after the laft experiment, the two rods are lafhed together near the bottom with a bit of thread, and then the ratchets unpinned and removed; on cutting the thread with a pair of fciffars they will each of them rebound, C towards M, and D towards N ; and if they rebounded refpectively to z and y, the mechanical power exerted would be the fame as it was after the ftroke, when the mean of their two afcents was up to the mark z ; but here it is not to be expected, becaufe not only the motion loft by the friction of the ratchets is to be deducted, becaufe it had the effect of real non-elafticity ; but alfo the elafticity that feparated them in the ftroke, which was loft in the vibrations that fucceeded; neither of which hindered the mean afcent to be to z ; but yet, under all thefe difadvantages in the machine (if not unreafonably ill made) the rod *ef* will afcend to *d*, and *gb* to *a* : and hence I infer, as a pofitive truth, that in the collifion of non elaftic foft bodies, *one half of the mechanic power refiding in the ftriking body is loft in the ftroke.*

Refpecting bodies unelaftic and perfectly hard, we muft infer, that fince we are unavoidably led to a conclufion concerning them, which contradicts what is efteemed a truth capable of the ftricteft demonftration ; *viz.* that the velocity of the center of gravity of no fyftem of bodies can be changed by any collifion betwixt one another, fomething muft be affumed that involves a contradiction. This perfectly holds,

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according

according to all the eftablifhed rules, both of perfectly elaftic and perfectly non-elaftic *foft* bodies ; rules which muft fail in the perfectly non-elaftic *hard* bodies, if their velocity after the ftroke is to the velocity of the ftriking body as one is to the fquare root of 2 ; for then the center of gravity of the two bodies will by the ftroke acquire a velocity greater than the center of gravity the two bodies had before the ftroke in that proportion, which is proved thus.

At the outfet of the ftriking body, the center of gravity of the two bodies in our cafe will be exactly in the middle between the two ; and when they meet it will have moved from their half diftance to their point of contact, fo the velocity of the center of gravity before the bodies meet will be exactly one half of the velocity of the ftriking body ; and, therefore, if the velocity of the ftriking body is 2, the velocity of the center of gravity of both will be one. After the ftroke, as both bodies are fuppofed to move in contact, the velocity of the center of gravity will be the fame as that of the bodies ; and as their velocity is proved to be the fquare root of 2, the velocity of their center of gravity will be increafed from 1. to the fquare root of 2. ; that is, from 1. to 1.414, &c.

The fair inference from thefe contradictory conclufions therefore is, that an unelaftic hard body (perfectly fo) is a repugnant idea, and contains in itfelf a contradiction ; for to make it agree with the fair conclufions that may be drawn on each fide, from clear premifes, we fhall be obliged to define its properties thus : that in the ftroke of unelaftic hard bodies they cannot *poffibly lofe any* mechanic power in the ftroke ; becaufe no other impreffion is made than the communication of motion ; and yet they *muft lofe a quantity* of mechanic power in the ftroke ; becaufe, if they do not, their common center of gravity, as

above fhewn, will acquire an *increafe* of velocity by their ftroke upon each other.

In a like manner the idea of a *perpetual motion*, perhaps, at firft fight, may not appear to involve a contradiction in terms; but we fhall be obliged to confefs that it does, when, on examining its requifites for execution, we find we fhall want bodies having the following properties; that when they are made to *afcend* againft gravitation their abfolute weight fhall be *lefs*; and that when they *defcend* by gravitation (through an equal fpace) their abfolute weight fhall be greater; which, according to all we know of nature, is a *repugnant* or *contradictory idea*.
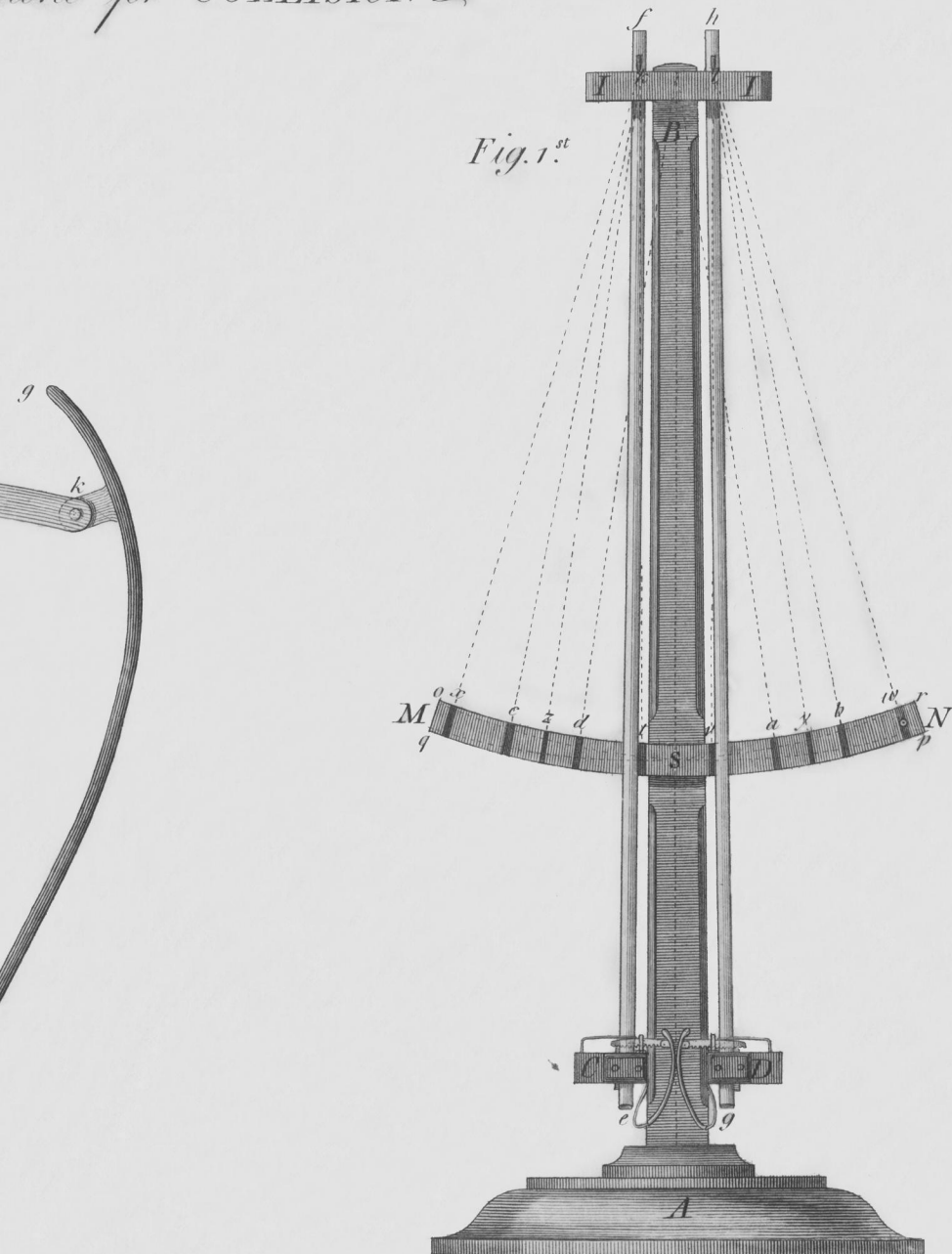
Machine for co...

Fig. 2.^d

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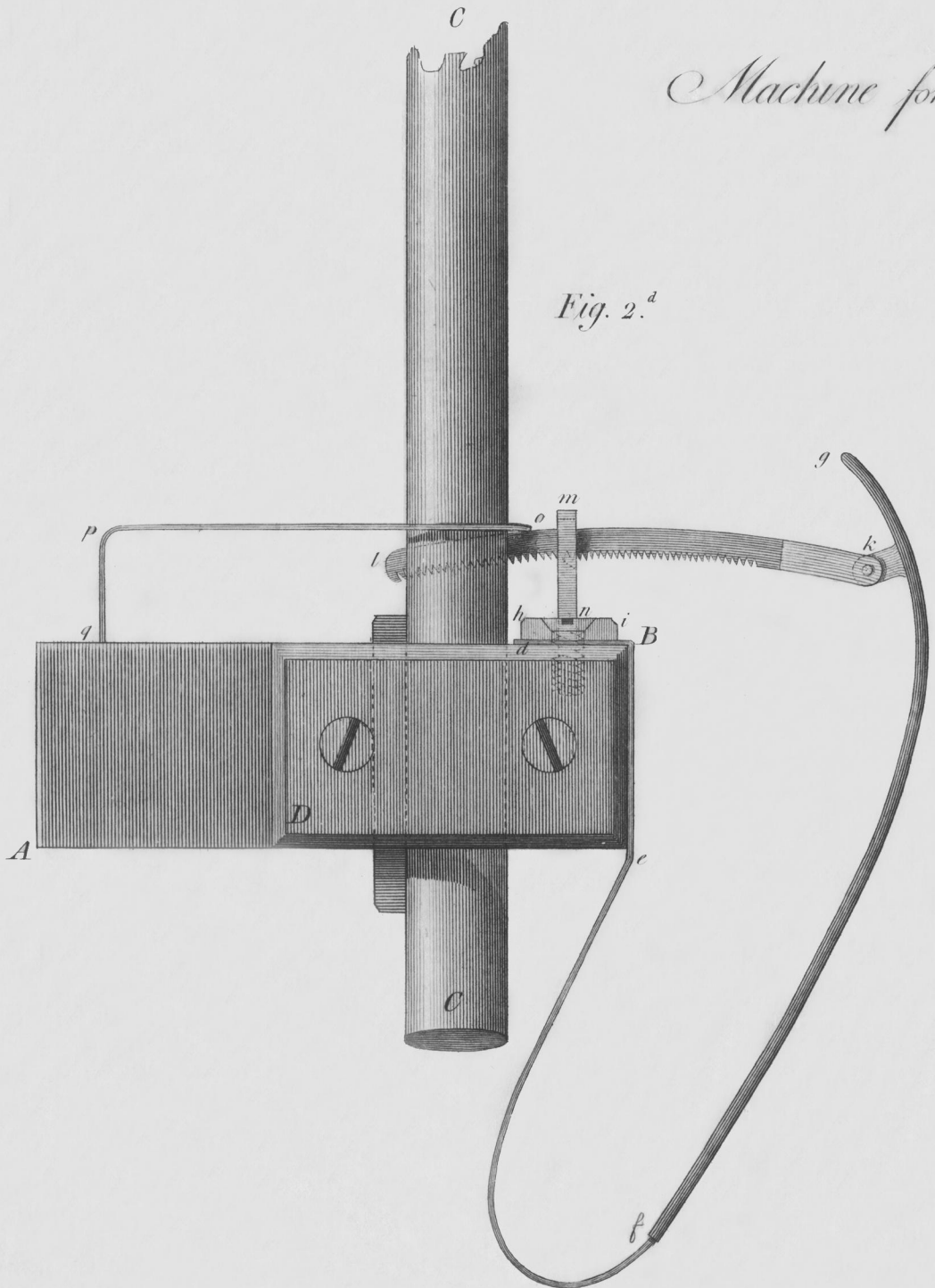
hine for COLLISION.

Fig. 1.ˢᵗ

Scale of Feet.